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USE OF GADGET IN RAIN OR FOG

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ABSTRACT

The loss of performance of a gadget exploded in rain or fog is considered. The losses for different charge weight equivalents W are proportional to a low power of W. According to one approximate solution, the power of W is 0.075; according to second, the power is 0.125. Using the experimental fact that the loss of peak pressure in the blast from 500-lb MC bombs at the 5 p.s.i. level in a fine wetting rain at Millersford, England in August 1944 was 5 percent, and the loss of positive duration was also 5 percent, it follows that the loss of performance for a gadget in similar conditions would be at least 15 percent in peak pressure. In an unusually heavy rain or dense fog, the concentration of liquid water in the air could be five times greater than it was at Millersford. The linear proportionality between loss and concentration of water would then break down. The solution to the next order of approximation is considered, and it is shown that the loss in peak pressure might be as high as 40 percent, and the loss in A and B damage area 30 percent. The loss of performance will vary with the air temperature, and on a cold winter's day will only be about one half of that as a hot summer's day.

USE OF GADGET IN RAIN OR FOG

Teller and Critchfield in some unpublished calculations deduced that a large explosion occurring in a rain or fog would lose considerably in power because of attenuation resulting from the evaporation of the water droplets. Similar calculations made independently are described in this report. Teller and Critchfield attempted to estimate the evaporation, and this, of course, is a formidable task; the results inevitably are uncertain. However, experimental evidence on bombs exploded in rain has very recently been obtained, and the problem of estimating losses for large explosions is thereby enormously simplified. All that is necessary is to establish scaling laws. The main part of this report is given to a consideration of the scaling laws from small to large explosions. The writer is indebted to Mr. Teller and Mr. Critchfield for discussions on this problem, and for permission to study their manuscript before completing this report.

PART I. QUALITATIVE ARGUMENTS ON 500-LB BOMB AND THE GADGET

There is definite experimental evidence that if a 500-lb MC bomb is exploded in fine rain, a loss of performance occurs. A number of these bombs (at least 12) were fired on successive days at Millersford (England), and two British teams (Armament Research Development, Woolwich and Road Research Laboratory, Department of Scientific and Industrial Research) made simultaneous recordings, each with their own sets of piezo-electric gauges, the purpose being a mutual check of consistency and absolute accuracy of recording. In every case, a check within one percent was obtained. However, the weather happened to be variable, and a fine rain was falling during some of the trials. The observation was made and confirmed by everybody concerned that the bombs were not giving their normal blast while rain was falling. The loss of positive blast impulse at radii where the peak overpressure was in the region 3-5 lb/in^2 was 10 percent and the loss of overpressure was 5 percent.

A moderate rain corresponds to about 0.1-cm precipitation per hour; the radii of the raindrops range from 0.02 to 0.2 cm. The proportion of liquid water to air by volume is about one part in ten million. A very heavy rain could correspond to a ratio five times higher. A very heavy country fog corresponds with a ratio of one part in a million. (See Brunt, Physical and Dynamical Meteorology.)

The proportion by volume of rain drops to air in the Millersford trials, mentioned above, was about one in ten million. The peak overpressure from a 500-lb MC bomb is 5 lb/in^2 at 35 metres. The volume of the raindrops in a hemisphere of radius 35 metres was thus 9×10^3 cc, and the heat of evaporation of this quantity of water is 5.5×10^6 calories. The energy in the positive blast wave at 35 metres is 6×10^6 calories, and in the rain is 10 percent less, namely 5.4×10^6 calories. Hence it is clear that not all of the raindrops in the 35-m hemisphere were evaporated; actually about one ninth of the water was vaporised. No doubt, the raindrops near to the explosion were completely evaporated, while further away, only partial evaporation occurred.

Suppose that, with the 500-lb bomb, complete evaporation occurred up to radius R_o. Then in an explosion n^3 times larger, (so that all linear dimensions and times in dry air would be scaled n times) complete evaporation up to radius $n R_o$ will certainly occur. Furthermore, it may be asserted with confidence that the loss in performance will increase practically linearly with the proportion by volume of liquid water in the air, provided the raindrops are of the same size and the loss of performance not too great. Thus, if the gadget is exploded in a heavy rain, or a thick fog, corresponding with a proportion by volume of water to air of one in two million, the peak pressures will be down by approximately

25 percent, and the areas of damage down about the same or a little less. These are not pessimistic estimates; in practice, an even more serious loss will occur, because the longer duration of the blast wave from the gadget will cause relatively more evaporation. There are fairly convincing arguments, given below, which indicate that the losses might be even twice as great.

Especially dangerous from the point of view of performance will be a dense fog, because the liquid water is already very finely divided and is readily evaporated by the blast wave. A heavy fine rain (Scotch mist) is almost equally dangerous for the same reason. A downpour (very large drops) may also be extremely effective in damping the blast wave, but the theory here does suffer from the disadvantage that it does not predict how long the large drops are able to remain intact in the blast. It seems very likely that they break almost instantaneously and if this is so, a downpour will be effective in destroying the power of the blast wave from a gadget.

PART II. MATHEMATICAL CONSIDERATIONS

We have suggested above that if a 500-lb bomb is exploded in a heavy rain or fog, the loss of performance may be a reduction of 20 percent in peak overpressure. We shall now show that, subject to the errors of a first-order calculation, and to errors in the calculation itself, the loss for a gadget may well be three or four times greater. Of course, as is usual with a first-order calculation, a correction of the order 80 or 100 percent is not to be taken seriously; higher-order calculations restore the position considerably.

The mechanism by which a blast wave is able to cause some evaporation of raindrops in the very short time in which it acts appears to be roughly as follows. A drop of water, past which air is flowing at speed U, experiences aerodynamical forces proportional to U^2. If the drop is to maintain its coherence, the surface tension (supported to a slight extent by internal viscous forces leading to surface tractions) must be sufficient to withstand the aerodynamical forces. The surface tensions are inversely proportional to the radius of the drop. Hence, approximately at any rate, if a drop of radius r is just stable in an air stream of velocity U, a drop of radius $r\,m^{-2}$ is just stable in an airstream of velocity $m\,U$. A drop of radius 0.33 cm. is just stable in an airstream of velocity 8 m/sec. (see Brunt, l.c.). Hence in an airstream of velocity 100 m/sec. (corresponding with a blast wave of overpressure 7.5 lb/in^2), the stable radius is only 0.0021 cm. Rapid disintegration of raindrops may therefore be expected as a blast wave of overpressure 7.5 lb/in^2 , or more, passes. The very fine droplets produced can evaporate to an appreciable extent in a millisecond. The energy dissipated in creating the droplets, i.e. the work done against surface tension and the work done in accelerating them, are negligible compared with the energy of the blast wave from a bomb or gadget (see Appendix I and II).

We therefore assume that when a blast wave passes a raindrop, instantaneous disintegration of the raindrop occurs, and that the resulting droplets are all of such size that they are just stable in the airstream behind the shock front. By this assumption, we are adopting a model in which more evaporation occurs than will actually occur in reality. The gain will be more for small explosions than for large, because the time lag in the creation of the droplets is relatively less important for blast waves of long duration. Therefore, if we use the experimental results on a 500-lb bomb to predict the results on a gadget, the errors caused by this assumption will be to underestimate the loss of performance of the gadget.

<u>Motion of a Droplet in An Airstream</u>

The equation of motion of a spherical droplet of constant radius r in an airstream of variable velocity u is

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$$(4/3)\pi r^3 \rho_w \, dv/dt = (1/2)\pi r^2 \rho_a (u-v)^2 \, C_D \, , \qquad (1)$$

where ρ_w and ρ_a are the densities of water and air, v is the velocity of the droplet and C_D is the drag coefficient. Referring to Goldstein (Modern Developments in Fluid Dynamics, Vol. 2, page 493), it will be seen that C_D is a function only of Reynolds number

$$R = (u-v)r/\nu \quad , \qquad (2)$$

where ν is the coefficient of kinematic viscosity of air, of numerical value 0.14.

For a blast wave, neglecting the suction phase, we may write

$$u = u_o \, e^{-\alpha t} \quad , \qquad (3)$$

while in accordance with the previous subsection, the radius of the just-stable droplet in an airstream u_o is

$$r = 0.33 \, (800/u_o)^2 \text{ cm} \qquad (4)$$

Let us consider a blast wave of peak overpressure 12 lb/in^2, this being outside the region of complete evaporation of a 500-lb bomb, but nevertheless near enough for appreciable evaporation to occur. The $u_o = 1.4 \times 10^4$ cm/sec, $r = 10^{-3}$ cm (a common radius in fog) and R initially is about 100. We wish to study the motion of the droplet. Referring to Goldstein (l.c. page 493) once again, it will be noticed that for $R < 1$, Stokes' law is valid for the drag. Hence a numerical solution of (1) over the region $100 \geq R < 1$ can be continued analytically for $R < 1$, to give solutions as far as desired.

Equation (1) is conveniently solved numerically by the following substitutions

$$T = \alpha t, \quad X = (u-v)/u_o \quad ,$$

so that X represents the differential velocity of the air and the droplet, expressed in terms of the initial value. Then (1) becomes

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$$dX/dT = -e^{-T} - \frac{3u_o\rho_a}{8r\alpha\rho_w} \quad X^2 \; C_D \tag{5}$$

with the boundary condition

$$T = 0, \; X = 1 \tag{6}$$

Goldstein plots experimental values of $\log_{10} \; C_D$ against $\log_{10} R$; since the initial value of R in our example is 100, and R is proportional to X, Goldstein's curve can be used immediately if the entries of $\log_{10} R$ are reduced by 2, and then called $\log_{10} X$. Numerical solutions over the range $100 \geq R > 1$, corresponding with $1 \geq X > 0.01$ were found for the 500-lb M6 bomb and the 1000-ton gadget, using $\alpha = 170 \; sec^{-1}$ for the former and $\alpha = 8.5 \; sec^{-1}$ for the latter.

At values of $R < 1$, we have Stokes' law

$$C_D = 24/R \tag{7}$$

and the equation of motion is

$$dX/dT = -e^{-T} - mX \tag{8}$$

where

$$m = 0.09 \; u_o\rho_a/r\alpha\rho_w \tag{9}$$

The solution is

$$X = (X_1 + \left[1/(m-1)\right] \; e^{-T_1} \;) \quad e^{-m(T-T_1)} - \left[1/(m-1)\right] \; e^{-T} \tag{10}$$

where T_1 is the value of T corresponding with $X = X_1$, the stage at which it may be assumed that Stokes' law becomes valid (actually, a convenient value of X_1 is 0.01 as mentioned above).

For the 500-lb MC bomb $m = 10$, and for the 1000-ton gadget $m = 200$. Hence in both cases for large T, the droplet moves faster than the airstream. It appears from the detailed solutions that X drops very rapidly from $X = 1$ at $T = 0$ to X of the order $\pm$ 3 percent in a very small fraction of the relaxation time $1/\alpha$. Fig. 1 shows how X varies with T, using a logarithmic scale for X, for the 500-lb MC bomb.

Fig. 1 could be amplified by including curves relating to the 1000-ton gadget. However, the proper curves to compare with those in Fig. 1 are those relating to "scaled" droplets. According to the hypothesis that "air flow" is the overriding factor in producing evaporation (see later sections), then the curves for the droplet at the edge of the complete evaporation zone are the same for the 500-lb bomb and the gadget; according to the "diffusion" hypothesis, m must be scaled up by $n^{1/2}$ for the gadget, the effect of which is to make the droplet more nearly follow the air motion. However, the variations are only in details of such a type that they do not much affect the arguments to follow.

The Relative Importance of Differential Speed, Thermal Conduction and Diffusion

The numerical results of the integration of (1) show that although the droplet very speedily acquires a velocity nearly equal to that of the air current, its relative velocity over the time $o < t < 1/\alpha$ is only for a very small part of this interval less in absolute magnitude than 3 percent of the initial air velocity. In the region of substantial, but not complete, evaporation the differential speed may be considered to be never less than 30 cm/sec.

The question arises whether the evaporation may be calculated as if the differential speed did not exist, i.e. as if the surrounding air were still.

As far as the relative speeds of thermal conduction and diffusion of water molecules away from the droplet are concerned, we may say that they are about equally important, so that neither may be neglected. The general justification for this statement is that the diffusion and heat transfer equations are formally identical, and the coefficient of thermal diffusivity k_1 is approximately equal, but actually a little less than the coefficient of diffusion of water molecules into air k_2. Numerical values are

$k_1 = 0.18 \qquad k_2 = 0.24$

The following argument shows that the heat reaching the droplet by thermal conduction will be appreciably different if the surrounding air is still or is moving at the approximate differential speed mentioned above, say 30 cm/sec. A water droplet of radius r in air at 100°C cannot chill the air near it to lower than the initial air temperature (before the blast wave arrives), say 10°C because the air was saturated at this temperature. It would need a sphere of still air of radius 18 r to be chilled to 10°C if enough heat were to be provided to evaporate the droplet.

At a point in hot still air imagine a negative point source of heat instantaneously introduced of just sufficient strength to evaporate the droplet. The spread of "cold" from this source is a well-known function. The drop in temperature $\Delta\theta$ at radius x at time $t = y/4k$ is given by

$$\Delta\theta = Q \frac{e^{-x^2/y}}{(\pi y)^{3/2}} \quad , \qquad (11)$$

where Q is the volume integral of the temperature due to the source. The time at which one-half of the "cold" has spread beyond a radius R is given by the equation

$$\int_0^R x^2 e^{-x^2/y} dx = 0.5 \int_0^\infty x^2 e^{-x^2/y} dx$$

the approximate solution of which is

$$R = \sqrt{y} \quad \text{or} \quad t = R^2/4k \quad . \qquad (12)$$

Hence, as far as order of magnitudes are concerned, we may say that half the heat of evaporating a droplet of radius r has been extracted from a sphere of radius 18r in time $324r^2/4k$. For a droplet of radius 10^{-3} cm this time is half a millisecond. Dividing the radius of this sphere by the time we get 36 cm/sec, exactly the same order of magnitude as the differential speed of the air and the droplet. However, the estimate of the speed of thermal conduction is certainly high, because the point source gives much larger thermal gradients than those really produced. We may therefore say that the evaporation of a droplet in still air will proceed considerably slower than the evaporation of a similar droplet when there is a differential speed between the droplet and the air. Two limiting cases, bracketing the true position, could be considered (1) still air, a lower limit (2) air motion so fast that the chilled air and water vapor are immediately removed, and upper limit. However, we shall not attempt to calculate the evaporation from first principles, but only obtain a scaling law to estimate the evaporation in a large explosion, knowing what it is with a 500-lb bomb. The scaling law is the same in both cases, and gives a lower limit to the evaporation in a large explosion.

The Scaling of the Evaporation Heat Conduction and Thermal Diffusion Only

Case I

Irrespective of the details of the mathematical solution of the evaporation problem, scaling laws exist which, together with the experimental information on the 500-lb MC bomb, provide us with the information which we seek on the gadget.

The first scaling law which is accurate is that if a droplet of radius a_1 and temperature T_o is placed in still air of a certain humidity and temperature $T_o + \phi$, its subsequent evaporation history will be identical with that of a droplet of radius a_2 and temperature T_o placed in the same still air, except that the time scale in the second case will be $m = (a_2/a_1)^2$ times longer. To prove this statement we note that the equation of thermal conduction and of molecular diffusion are both of the same formal type

$$\frac{\partial \psi}{\partial t} = k \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r}\right) \qquad (13)$$

where k has different values for the two cases. It is of course necessary to assume that the coefficient of thermal diffusivity is independent of humidity, and the coefficient of diffusion is independent of temperature.

The rates of flow of heat into the droplets are in the ratio

$$r_1^2 \ (\partial\psi/\partial r)_{r_1} \qquad \text{and} \qquad r_2^2 \ (\partial\psi/\partial r)_{r_2}$$

where ψ is the temperature at point r outside the droplet, and these are in the ratio $1 : \sqrt{m}$. These rates of flow will be causing changes of temperature, $\partial T_1/\partial t$ and $\partial T_2/\partial t$, and evaporation. The rates of evaporation may be measured by dr_1^3/dt and dr_2^3/dt, which are in the scaled ratio $1 : \sqrt{m}$, as required. The rates of change of temperatures with time are in the ratio (rate of heat flow into droplet : heat capacity) which are in the scaled ratio $m : 1$, as required.

The second scaling law is best expressed as an inequality. Suppose that, as in the first scaling law, we seek a connection between the evaporation of a droplet 1, initially at temperature T_o, placed in still air at temperature $T_o + \phi_1$, and a droplet 2, initially at T_o, placed in the same still air but adiabatically at a temperature $T_o + \phi_2$. We suppose droplet 1 smaller than droplet 2, time scale 1 less than a variable time scale 2, $\phi_1 > \phi_2$. We assume that the humidity of the air corresponds with the vapor pressure of water at T_o, and is negligible irrespective of the fact that the water vapor originally in the air has been compressed. This is a very good approximation because T_o is about 10°C, and the ϕ's are at least 50°C.

Both droplets become warmer, and since only small quantities of heat are needed to change their temperatures (as contrasted with evaporating them) we may suppose that in both cases a quasi-static regime obtains. Let $T_o + \theta_1$, be the temperature of droplet 1 at time t_1 and $T_o + \theta_2$ the temperature of droplet 2 at the corresponding time t_2.

The heat flowing into the droplet 1 $\sim r_1^2 (\phi_1 - \theta_1)$

The heat being used in evaporation $\sim r_1^2 f(\theta_1) = r_1^2 (\alpha\theta_1 + \beta\theta_1^2)$

where α and β are positive and are to be determined from the vapor-pressure curve. Since these balance

$$A(\phi_1 - \theta_1) = \alpha\theta_1 + \beta\theta_1^2$$

where A is positive

$$\phi_1 = (1 + \frac{\alpha}{A})\,\theta_1 + \frac{\beta}{A}\,\theta_1^2$$

Similarly

$$\phi_2 = -(1 + \frac{\alpha}{A})\,\theta_2 + \frac{\beta}{A}\,\theta_2^2$$

From the positive sign of all the coefficients in these equations, it follows that (θ/ϕ) is a decreasing function of ϕ, and is always less than unity. Approximate numerical values of θ, ϕ and θ/ϕ can be obtained from wet and dry bulb hygrometry. For air saturated at temperature 10°C, we find

Table I

ϕ	18.6	26.7	41.7	69.1
θ	13.6	16.7	21.7	29.1
θ/ϕ	0.42	0.40	0.37	0.32

Unfortunately, the experimental results do not allow the table to be continued to higher values of ϕ; since the region of substantial but incomplete evaporation corresponds to ϕ in the region of 50 to 100°C, we see that $1 - \theta/\phi$ in the region of interest lies within a few percent of 0.66.

The scaling law is

$$\frac{\delta t_1}{\delta t_2} \simeq \frac{r_1^2(\phi_2 - \theta_2)}{r_2^2(\phi \text{[illegible]}} \quad (14)$$

where δt_1 and δt_2 are corresponding infinitesimal time intervals, t_2 being chosen so that ~~t_2/t_1~~ τ_2/τ_1 $= a_2/a_1$.

Writing this in the form

$$\frac{\delta t_1}{\delta t_2} = \frac{a_1^2 \phi_2 (1-\theta_2/\phi_2)}{a_2^2 \phi_1 (1-\theta_1/\phi_1)} \qquad (15)$$

and using the fact that θ/ϕ, which, of course, is positive and less than 1, is a decreasing function of ϕ, we see that

$$\frac{\delta t_1}{\delta t_2} > \frac{a_1^2 \phi_2}{a_2^2 \phi_1} \qquad (16)$$

This inequality applies to still air, the temperature of the air not being changed except by the droplets themselves. If the droplet is in a blast wave, then ϕ_1 and ϕ_2 are varying, but the assumption of quasi-static conditions, which is a reasonable approximation, then fixes $t_2/t_1 = n$. Furthermore, in the region of nearly complete to very small evaporation, the overpressure p varies like $R^{-2.67}$, where R is the distance from the center of explosion. Let R_1 and R_2 be corresponding radii of evaporation in a small explosion and one n^3 times greater. Then the inequality (16) gives us that

$$n < (p_2/p_1)^5$$

or

$$R_2/nR_1 > n^{0.075} \qquad (17)$$

The energy of evaporation for the large explosion, per unit weight of explosive charge, is therefore greater than $n^{0.225}$ times that for the small explosion.

Comparing a 500-lb bomb with a 1000-ton gadget, and assuming a 5 percent loss of peak pressure with the bomb, we find greater than 10 percent loss with the gadget. The loss for a 10,000-ton gadget is greater than 12 percent.

These figures also give approximately the loss in areas of A and B damage. The losses for all of these weapons would be appreciably greater in heavy rain or fog.

Evaporation in a Fast Air Stream

Case II

Here the relative air stream is assumed to sweep away the water molecules and the cooled air before any concentration is built up. The speed of the evaporation is greatly increased but the sealing laws of the previous section are not changed, and the estimated losses for gadgets are unaffected.

The Law of Variation of Evaporation with Distance. A Second Sealing Law.

As the blast wave expands, it breaks the raindrops into droplets, the size of which increases rapidly with distance. Up to a certain radius R_o, depending on the size of the explosion, the droplets are completely evaporated. From this radius outwards, the amount of evaporation decreases rapidly, and it is the purpose of this section to estimate the rate of decay, and to make deductions therefrom.

Let r be the radius of the droplet, τ the duration of the positive blast wave, ϕ the initial temperature at the shock wave front above the initial temperature of the droplet (and therefore above the temperature at which the air would be saturated, as previously but not quite accurately, assumed).

Then the amount of evaporation occurring at any radius R (where droplets still exist) is proportional to

$$r^2 \tau \phi (1-\theta/\phi) \qquad (18)$$

This may be written

$$r^2 \tau \phi \left[1 - G(\mu)\right] \qquad (19)$$

where $\mu = R/R_o$, and R_o is a length defining the scale of the explosion. The function $G(\mu)$ is positive, less than 1, and increases with increasing

μ to a limiting value, and for ϕ in the region 40-80°C , G is about 2/3 (see Table 1).

For the moment, let us replace r in this expression by a, the initial radius of the droplet. The consequences of this substitution will be investigated below.

The density of the droplets is proportional to a^{-3}. Hence by the present assumption, the amount of evaporation at R is proportional to

$$a^{-1}\tau\phi\left[1-G(\mu)\right] \quad , \qquad (20)$$

and is therefore proportional to

$$R^{-8}\tau\left[1-G(\mu)\right] \qquad (21)$$

Clearly, the effect of replacing r by a is to make the evaporation decrease too fast with R. This is because at larger R the droplets are larger, evaporation does not decrease their radius proportionately as much as for the smaller droplets nearer in, and their surface area, from which evaporation occurs, is not decreased so much. The variation of τ with R is between $R^{1/2}$ and R ; this is so weak compared with R^{-8} that we neglect it.

Fig. 2 shows diagrammatically the evaporation as a function of radius in two cases. For a small explosion, evaporation is complete up to R_o , and then decreases rapidly, approximately like $(R/R_o)^{-8}$. For an explosion n^3 times larger, evaporation is complete to radius nR_o' , where R_o' is greater than R_o because the longer duration of the blast wave succeeds in evaporating relatively more droplets. To calculate R_o' , we have the relationship

$$n < (R_o'/R_o)^8 F \qquad (22)$$

where

$$F = \left[1-G(1)\right] / \left[1-G(R_o'/R_o)\right] \qquad (23)$$

[redacted]

Now F is an extremely weakly varying function, as follows from Table I, and the inequality cannot be upset by replacing F by unity, although the true value is slightly less than unity. Hence

$$R_o' > R_o \, n^{1/8} \qquad (24)$$

The loss of energy due to evaporation in the explosion n^3 times greater is therefore $n^{0.375}$ greater per unit weight of charge. Scaling up the 5 percent loss in peak overpressure of the 500-lb MC bomb in a moderate rain, gives a loss of 16 percent in peak overpressure for a 1000-ton gadget and a loss of 22 percent for a 10,000-ton gadget. The predicted losses in heavy rain or fog are correspondingly greater, perhaps by a factor 3 or more. It must be pointed out that these estimates are <u>lower limits</u> in a first-order theory. We must now consider the correction to this theory in heavy rain or fog, because the position is somewhat restored by this calculation.

Correction to First-Order Theory

So far, we have made two assumptions:

1) The evaporation has not been so great that it was necessary to make an allowance for the attenuation of the blast wave due to evaporation, in estimating the evaporation.

2) The blast wave substantially preserves its form, but the peak pressure and the positive duration are reduced.

Let us extend (2) and incorporate the experimental result that the percentage loss of peak pressure is roughly equal to the percentage loss of positive duration. This assumption is reasonable, but difficult to justify theoretically. The evaporation is a "volume" effect; energy is being absorbed from the positive pulse and the resulting cooling shortens the pulse. The non-linear nature of the

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propagation causes the effect of the energy losses behind the front to spread to the front, and thus reduce the peak pressure. Further investigation, of a difficult nature might reveal that the relative losses of peak pressure and duration are about equal, but for the moment we rely on the experimental result that this is so.

We now state the experimental results for blast waves for bombs in dry air. From logarithmic plots the following decay laws appear to be the best simple ones. In the region 20-6 lb/in^2 , the energy in the blast per unit area varies like $R^{-8/3}$; the positive impulse per unit area varies like $R^{-4/3}$; and the peak overpressure varies like $R^{-8/3}$. It is clear from these statements that the form of the wave does change a good deal in the region under consideration.

According to previous sections, the heat of evaporation of rain per unit volume in the first-order calculations was proportional to R^{-8} , provided the evaporation was not complete. It is therefore proportional to the cube of the blast energy of the positive pulse per unit area. We take this result over into a second-order calculation. The amount of evaporation is conditioned by the peak pressure P, and is proportional to P^3. In the region considered, P is proportional to the energy of the positive pulse per unit area. Of course, the energy in the negative part of the pulse is probably negative, and at greater radii, the energy of the whole pulse ultimately becomes proportional to the square of P (sound theory).

Accepting the above paragraph as correct, it is possible, as shown later, to calculate the effect of the rain on the attenuation, allowing for the attenuation that would occur in the absence of rain. However, what we are mainly interested in is damage area. The criterion for damage from a small bomb, such as 500 lb, is certainly impulse, but for large explosions such as gadgets, the criterion is peak pressure. We are therefore faced with the problem of estimating the loss in

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peak pressure when the loss of energy is known. The following statement appears the best decision that can be made in our present state of knowledge. If the energy of the positive blast wave is reduced by a factor j, then the peak pressure and the positive duration are both reduced by $j^{1/2}$, while the positive impulse is reduced by j. These estimates are on the favorable side, and the real reduction factor on pressure is probably less. At the radius of transition from A to B damage, say 6 lb/in^2, practically no evaporation occurs. We therefore, assume the above decay laws. Thus, the reduction factor for any level of peak pressure is $j^{3/16}$, and the reduction factor in area is $j^{3/8}$.

Attenuation Calculations. Let W_o be the total energy of the blast wave crossing the surface of a sphere of radius R_o, the air being dry (no liquid water). Suppose that R_o is the radius of complete evaporation of a rain or fog according to the first-order theory, and that the heat of evaporation of the rain inside R_o is qW_o. Let ε_c be the energy of complete evaporation of rain per unit volume.

Then we have

$$(4\pi/3)\ R_o^3\ \varepsilon_c = qW_o$$

or

$$\varepsilon_c = 3qW_o/4\pi R_o^3 . \qquad (25)$$

The energy of evaporation to infinity, according to the first-order theory is

$$4\pi\varepsilon_c \left[\int_0^1 x^2 dx + \int_1^\infty x^{-8} x^2 dx \right] = 8qW_o/5 \qquad (26)$$

The equation of energy, taking cognizance of the fact that W varies like $R^{-2/3}$ in the absence of rain, is

$$\frac{dW}{dR} + \frac{2}{3}\ \frac{W}{R} + 4\pi R^2 \varepsilon_c = 0 \qquad (27)$$

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The solution is

$$W = W_o(R_o/R)^{2/3} \left[1-(12\pi\varepsilon_c R_o^3/11\ W_o)\ (R/R_o)^{11/3}\right] \qquad (28)$$

Now ε_c is determined already by the assumptions made in the first-order theory, and is given by (25). Hence

$$W = W_o\ (R_o/R)^{2/3} \left[1-(9q/11)\ (R/R_o)^{11/3}\right] \qquad (29)$$

Complete evaporation is occurring when the energy flow is greater or equal to $(W_o/4\pi R_o^2)$ per unit area. Hence the actual radius of complete evaporation is given by

$$\lambda^{8/3} = 1 - 9q^{11/3}/11 \qquad (30)$$

where

$$\lambda = R/R_o\ .$$

The equation of energy outside the region of complete evaporation is

$$\frac{dW}{dR} + \frac{2W}{3R} + 4\pi R^2\varepsilon = 0 \qquad (31)$$

where ε is the energy of evaporation per unit volume at R.

But

$$\varepsilon = k(WR_o^2/R^2W_o)^3\ , \qquad (32)$$

where the constant of proportionality is given by the first order calculation, namely

$$W = W_o\ ,\ \varepsilon = \varepsilon_c \quad \text{when } R = R_o\ .$$

Hence

$$k = \varepsilon_c \qquad (33)$$

Therefore, writing

$$y = W/W_o\ ,\ x = R/R_o\ ,$$

the equation of decay takes the non-dimensional form

[redacted]

$$x^4(dy/dx) + 2/3\,(x^3\,y) + 3q\,y^3 = 0 \qquad (34)$$

Writing

$$Y = x^{2/3}y \quad ,$$

we find

$$Y^{-2} = L - (18q/13)x^{-13/3} \qquad (35)$$

where L is a constant of integration, determined for any q value from the solution of (30), namely when

$$x = \lambda\ ,\ y = \lambda^2\ ,\ Y = \lambda^{8/3}$$

so that

$$L = \lambda^{-13/3}\left[\lambda^{-1} + (18q/13)\right] \qquad (36)$$

Hence

$$y = x^{-2/3} \Big/ \left[L - (18q/13)x^{-13/3}\right]^{1/2} \quad , \qquad (37)$$

a formula from which the energy at any radius x may be found. In particular, the peak pressure 6 lb/in^2 occurs at about x equal to 2; for this value of x, the term in $x^{-13/3}$ in the denominator of (37) is negligible, the reason being that no more evaporation is occurring.

The law of decay if no rain were present is

$$y = x^{-2/3} \qquad (38)$$

Hence at the region where A and B damage meet, we have that the reduction in blast energy is

$$L^{-1/2} \qquad (39)$$

In the case of very small values of q, it will be found that very nearly

$$L = 1 + 3q \qquad (40)$$

so that the percentage loss of blast energy is 150 q.

The q value corresponding with the 500-lb bombs at Millersford was 0.067.

The table below gives a few representative values of the solutions of the above equations for special q values.

Table II

q	λ	L	$L^{-1/2}$	$L^{-3/16}$	Rain
0	1.0	1.0	1.0	1.0	None
0.067	0.98	1.24	0.90	0.96	v. light
0.20	0.945	1.71	0.76	0.90	-
0.333	0.919	2.23	0.67	0.86	Millerford
0.394	0.906	2.56	0.63	0.84	-
.50	0.890	3.10	0.57	0.81	Moderate
.70	0.864	4.13	0.49	0.76	-
1.00	0.829	5.83	0.41	0.71	Heavy

The column headed "rain" gives a rough indication of the intensity of rain for gadgets in the range 1000 to 10,000 tons. The column headed $L^{-1/2}$ gives the reduction in blast energy, and the column $L^{-3/16}$ gives the reduction in A and B damage areas, both as upper limits (i.e. the actual reduction factors are smaller than those given here, and the corresponding losses in performance greater).

The concentration of liquid water by volume in the air corresponding with any q value is 1 part in N million, where

1) $N = 2.13/q$ for a gadget of 1000 tons

2) $N = 2.93/q$ for a gadget of 10,000 tons

3) $N = 0.667/q$ for a bomb 500 lb.

One part in 2 million is a heavy rain or fog; one part in 5 million is a moderate rain or fog.

Anomalous Effects

Variations in the performance may reasonably be expected, even when the concentration of liquid water in the air is kept constant. The following factors are to be considered.

1) Variations in the size of the raindrops. Here the time of breakup into small droplets is unknown, and hence the amount of evaporation might depend on the size of the raindrops. As already explained, an effect of this type will cause the loss of performance of gadgets to be greater than estimated above.

2) Variations in air-temperature. On a winter's day, the air temperature, while rain is falling, might be just above 0°C ; in a blast wave of 7.5 lb/in², this is raised to 34°C. On a summer's day, the air temperature in rain might be 27°C ; in a blast wave of 7.5 lb/in², this is raised to 65°C. The loss of performance due to a given rain may therefore be twice as great in summer as in winter. It is satisfactory to note that the Millersford trials, upon which the estimates made in this report are based, were made in summer, and that it is therefore unnecessary to increase our estimates of possible losses because the air temperatures might be considerably higher than they were at Millersford.

[redacted]

APPENDIX I

Only an inappreciable amount of energy is absorbed from the blast wave in giving kinetic energy to the raindrops. The concentration by volume of water to air is less than 10^{-6} , or say 10^{-3} by mass. If all the raindrops inside a blast wave of peak overpressure 5 lb/in^2 had the full velocity of the air at the front of the blast wave, the kinetic energy of the raindrops would be about 0.01 of the kinetic energy of the air in the positive part of the pulse. This in turn is less than one-half of the energy of the pulse.

APPENDIX II

Only an inappreciable amount of energy is absorbed from the blast wave in breaking up the raindrops into fine droplets. Suppose a raindrop of radius 0.02 cm is placed in an airstream 1.45×10^4 cm/sec (peak overpressure of 12 lb/in^2). Then 8,000 droplets, each of radius 10^{-3} cm are created. The work done against surface tension is 8 ergs per raindrop. Assuming the air carries one part in a million by volume of raindrops, the work done against surface tension in breaking up all the raindrops inside the blast wave at the stage where the peak overpressure is 12 lb/in^2 has an equivalent of 7×10^{-6} calories per gram of charge. The positive blast wave itself has an energy content of about 50 calories per gram of charge.

[redacted]

DESCRIPTION OF FIGURES

Fig. I. The figure shows the logarithm to base ten of X, the differential speed of a droplet of constant radius 10^{-3} cm and the air in the blast wave from a 500-lb MC bomb at the radius at which the peak overpressure is 12 lb/in^2, the unit of velocity being 140 m/sec, the initial mass velocity of the air behind the shock wave. It will be seen that the droplet very quickly picks up speed, and that at the time T = 0.16 α , where α is the relaxation time for the blast wave (6 milliseconds) the differential speed is zero. Thereafter the droplet is moving faster than the air. The differential speed is less than 3 percent of the initial air speed for only 0.078 α i.e. 0.47 milliseconds.

Fig. II. This figure shows diagrammatically the percentage evaporation as a function of radius in a small explosion, and the "scaled down" evaporation in an explosion n^3 times greater. The scaled radius of complete evaporation in the large explosion is greater than the radius of complete evaporation in the small explosion because the duration of the blast is longer in the former case.

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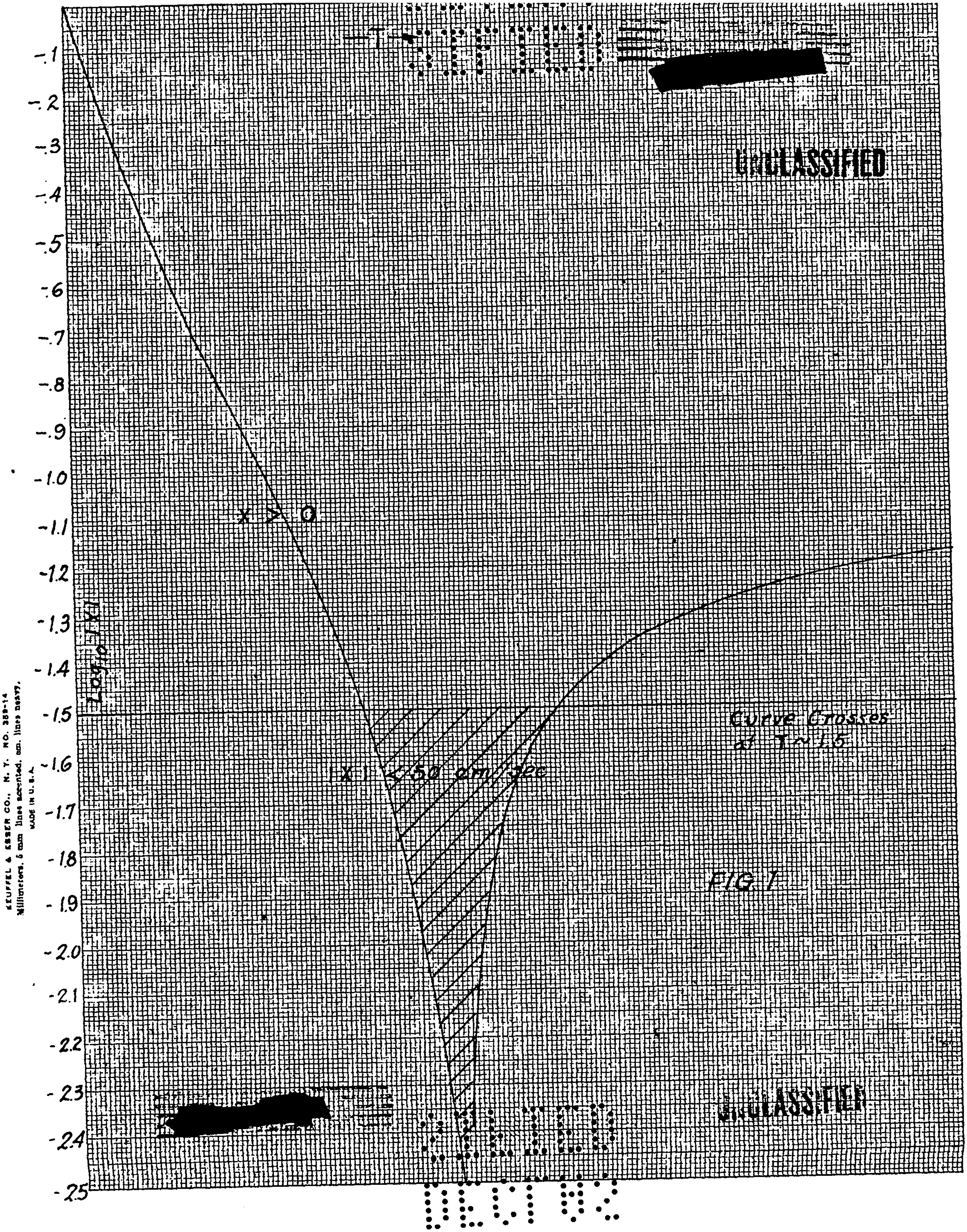

FIG. 1

[redacted]

[redacted]

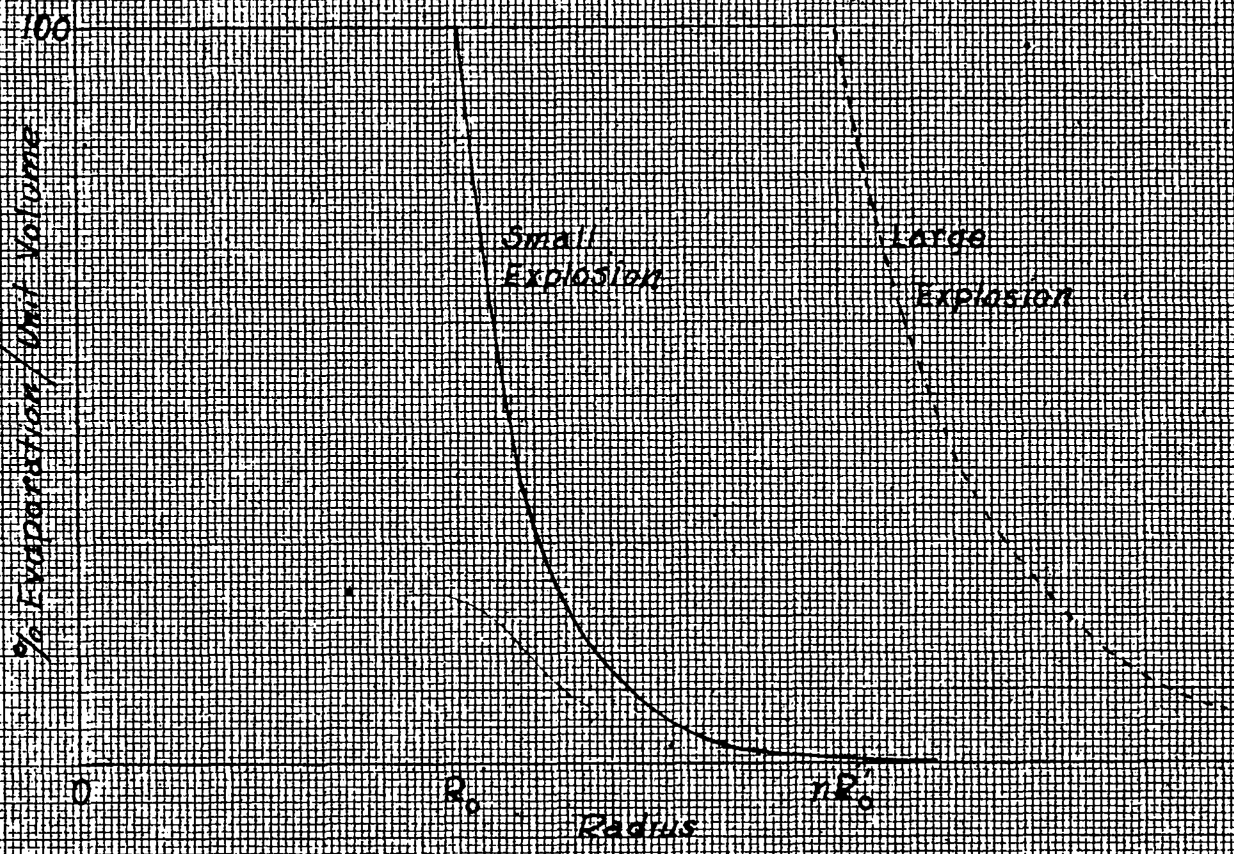

FIG. 2

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